

## Equation of a circle - completing the square

Consider the expansion of the squares  $(x+5)^2$  and  $(x-5)^2$ .

$$(x+5)^2 = x^2 + 10x + 25$$

$$5 \xrightarrow{\times 2} 10$$

$$5 \xrightarrow{\text{square}} 25$$

and

$$(x-5)^2 = x^2 - 10x + 25$$

$$-5 \xrightarrow{\times 2} -10$$

$$-5 \xrightarrow{\text{square}} 25$$

We need to reverse this process.

The coefficients of  $x^2$  and  $y^2$  each need to be 1.

If necessary, first divide both sides of the equation by a suitable number.

Consider the equation:

$$x^2 + 10x + y^2 - 6y + 18 = 0$$

Subtract 18 from both sides:

$$x^2 + 10x + y^2 - 6y = -18$$

Calculate the constant terms to complete the squares:

$$10 \xrightarrow{+2} 5 \xrightarrow{\text{square}} 25$$

$$-6 \xrightarrow{+2} -3 \xrightarrow{\text{square}} 9$$

Add 25 and 9 to both sides:

$$x^2 + 10x + 25 + y^2 - 6y + 9 = -18 + 25 + 9$$

Simplify:

$$(x+5)^2 + (y-3)^2 = 16$$

$$(x+5)^2 + (y-3)^2 = 4^2$$

The centre is  $(-5, 3)$  and the radius is 4.