MODELLING HOURS OF DAYLIGHT FOR 1999

Consider the data below.

Enter the data as lists $L_1 \rightarrow L_4$.

Use the regression feature to model the data by a sine function.

Comment on the data and the models.

		Amount of daylight (mins)		
date	day	Brisbane	Adelaide	London
1 Jan	1	831	868	481
1 Feb	32	803	829	553
1 Mar	60	761	772	655
1 Apr	91	710	704	776
1 May	121	664	643	886
1 Jun	152	631	598	973
1 Jul	182	625	590	988
1 Aug	213	649	623	922
1 Sep	244	692	680	814
1 Oct	274	740	745	700
1 Nov	305	789	810	584
1 Dec	335	824	859	496

BRISBANE

```
SinReg
 y=a*sin(bx+c)+d
 a=104.2832437
 b=.0165994891
 c=1.836016114
 d=730.1856397
```

ADELAIDE

```
SinReg
 y=a*sin(bx+c)+d
 a=141.0169661
 b=.016478486
 c=1.858862382
 d=732.7028831
```

LONDON

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SinReg
 y=a*sin(bx+c)+d
 a=253.7619356
 b=.0166754666
 c=-1.309013752
 d=728.0889192
```

MODELLING THE AMOUNT OF DAYLIGHT DURING 1999

The equations produced using the sine regression feature on the TI83 can be rearranged into the form $y = a\sin\left[\frac{2p}{\text{period}}(x-h)\right]+k$ where a is the amplitude, h is the phase shift (x direction) and k is the shift in the y direction.

BRISBANE

$\text{period} = 378.5$

$a = 104.3$

$h = {}^{-}110.6$

$k = 730.2$

$$\begin{aligned} y &= 104.3\sin(0.0165994891x + 1.836016114) + 730.2 \\ &= 104.3\sin\left[0.0165994891\left(x + \frac{1.836016114}{0.0165994891}\right)\right] + 730.2 \\ &= 104.3\sin\left[\frac{2p}{2p/0.0165994891}(x+110.6)\right] + 730.2 \\ &= 104.3\sin\left[\frac{2p}{378.5}(x+110.6)\right] + 730.2 \end{aligned}$$

ADELAIDE

$\text{period} = 381.3$

$a = 141.0$

$h = {}^{-}114.4$

$k = 732.7$

$$\begin{aligned} y &= 141.0\sin(0.016478486x + 1.8858862382) + 732.7 \\ &= 141.0\sin\left[0.016478486\left(x + \frac{1.8858862382}{0.016478486}\right)\right] + 732.7 \\ &= 141.0\sin\left[\frac{2p}{2p/0.016478486}(x+114.4)\right] + 732.7 \\ &= 141.0\sin\left[\frac{2p}{381.3}(x+114.4)\right] + 732.7 \end{aligned}$$

LONDON

$\text{period} = 376.8$

$a = 253.8$

$h = 78.5$

$k = 728.1$

$$\begin{aligned} y &= 253.8\sin(0.0166754666x - 1.309013752) + 728.1 \\ &= 253.8\sin\left[0.0166754666\left(x - \frac{1.309013752}{0.0166754666}\right)\right] + 728.1 \\ &= 253.8\sin\left[\frac{2p}{2p/0.0166754666}(x-78.5)\right] + 728.1 \\ &= 253.8\sin\left[\frac{2p}{376.8}(x-78.5)\right] + 728.1 \end{aligned}$$

The values for the periods are a little larger than the expected 365 days.

The phase shifts (h) broadly match the equinox dates of 21 March (day 80) and 23 September (day 266).

The graphs intersect at the equinox dates where the amount of daylight matches the shift in the y direction (k) of 730 minutes. At the equinoxes, all places on the earth's surface have the same amount of daylight of 730 minutes (12 hours and 10 minutes).

- http://www-spof.gsfc.nasa.gov/stargaze/Sseason.htm for equinox details.
- http://www.exptech.com/sunrise.htm for daylight calculations.

MODELLING THE AMOUNT OF DAYLIGHT IN BRISBANE DURING 1999

Here is a trigonometric function with a period of 365 days and a phase shift of 266 days (corresponding to the 23 September equinox):

$$y = 104\sin\left[\frac{2p}{365}(x-266)\right]+726$$

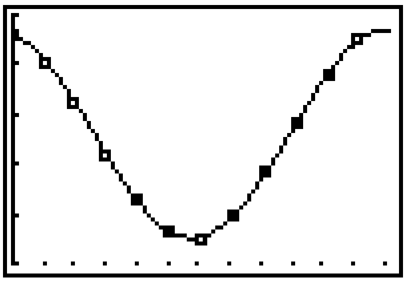
Does this equation model the data better than the equation produced by the TI83 sine regression feature?

Compare the original values with those produced by the two equations:

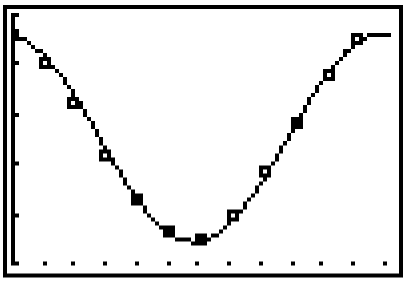
day	amount of daylight (minutes)	$y = 104.3\sin\left[\frac{2p}{378.5}(x+110.6)\right]+730.2$	$y = 104\sin\left[\frac{2p}{365}(x-266)\right]+726$
1	831	830	829
32	803	803	807
60	761	762	767
91	710	709	713
121	664	663	663
152	631	632	630
182	625	627	623
213	649	648	644
244	692	690	688
274	740	741	740
305	789	790	791
335	824	824	822

Compare the fit of the graphs to the original data:

$$y = 104.3\sin\left[\frac{2p}{378.5}(x+110.6)\right]+730.2$$



$$y = 104\sin\left[\frac{2p}{365}(x-266)\right]+726$$



Conclusion:

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