

- 1) The tide in a river has a variation from high tide to low tide of 60 cm. During a 24 hour period, there are two high tides and two low tides. The tidal patterns form a sinusoidal wave (sine or cosine)

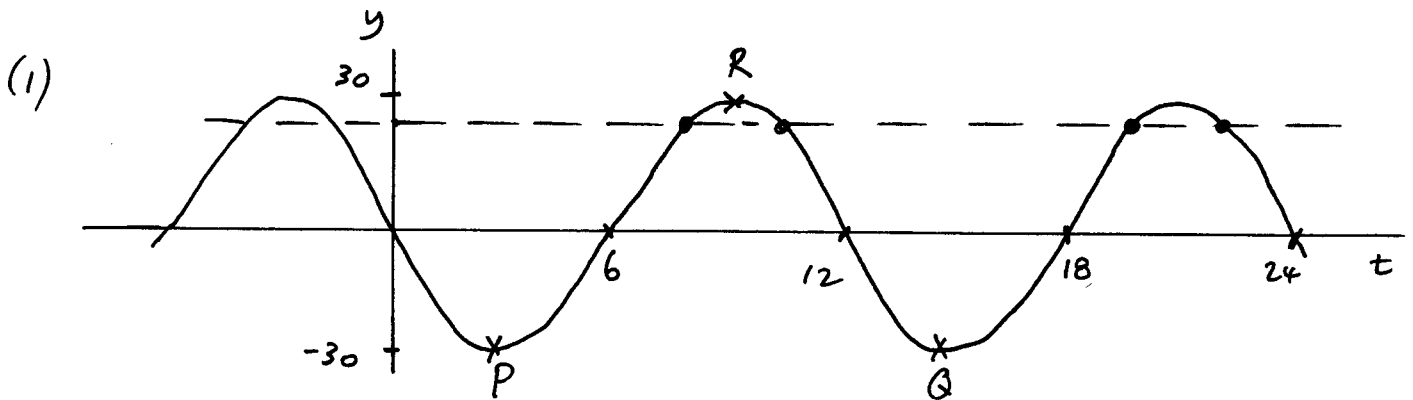
At midnight the tide is half way between high and low tide and receding.

- Give the trigonometric function to represent the tidal pattern
  - At what time(s) does low tide occur?
  - What period of time elapses between a high tide and a low tide?
  - A fisherman will only go fishing when the tide is 25 cm or more above the “average” tidal position. During what times in a day should he go fishing?
- 2) A pendulum swings in such a way that it’s displacement from the “middle position” or “bottom of the swing” is given by the formula,

$$x(t) = 14 \sin\left(2t + \frac{\pi}{2}\right)$$

$x$  is the displacement from the “middle position” centimetres  
 $t$  is time in seconds.

- What is the initial displacement of the pendulum?
  - When does the pendulum first reach the “middle position”?
  - How long does it take the pendulum to complete a full swing?
  - Can this function be written in a simpler form and if so, what is that form?
- 3) a) Use your calculator to draw the graph of  $y = 3 \sin x - 4 \cos x$
- b) Express  $y = 3 \sin x - 4 \cos x$  in the form
- $y = r \sin(x + \alpha)$
  - $y = r \sin(x - \alpha)$
  - $y = r \cos(x + \alpha)$
  - $y = r \cos(x - \alpha)$



- (a) Let  $y$  cm be the height above the half way mark.  
Let  $t$  hours be the length of time since midnight.

$$y = a \sin \frac{2\pi}{\text{period}} (t - h) + k$$

$$a = -30, \text{ period} = 12, h = 0, k = 0$$

$$\therefore y = -30 \sin \frac{2\pi}{12} t$$

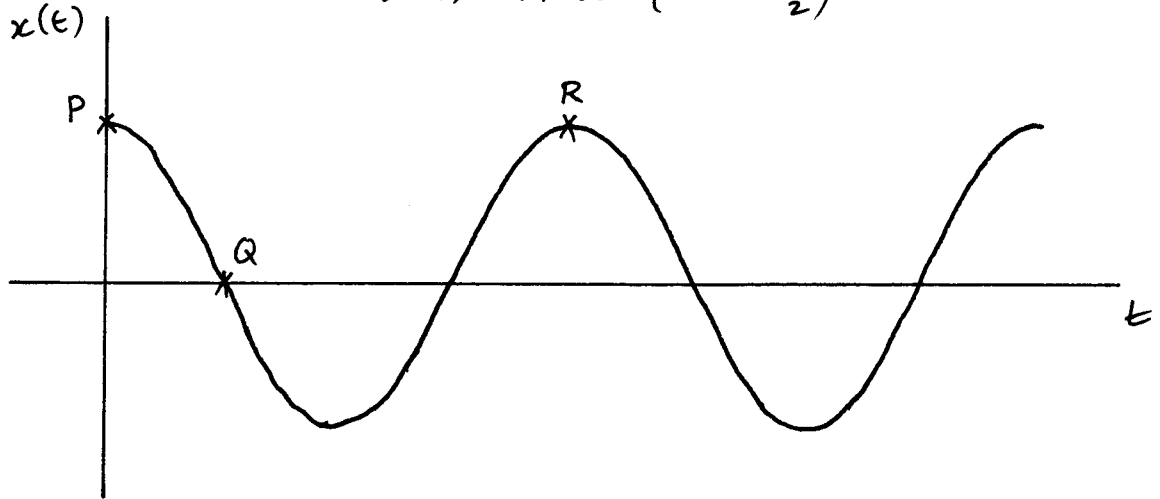
$$y = -30 \sin \frac{\pi}{6} t$$

- (b) Low tide occurs at 3.00 am and 3.00 pm.  
This is shown by points P and Q on the graph.
- (c) Time between a high tide and a low tide is 6 hours. See points R and Q on the graph.
- (d) Draw the graph  $y = 25$  and find the points of intersection with the tide graph.  
 $t \approx 7.8814, 10.1186$  and 12 hours later

$\therefore$  The fisherman should go fishing:  
from 7.47 am to 10.07 am  
and from 7.47 pm to 10.07 pm

(2)

$$x(t) = 14 \sin\left(2t + \frac{\pi}{2}\right)$$



Inspection of the graph on the TI-83 gives:

$$P (0, 14)$$

$$Q \left(\frac{\pi}{4}, 0\right)$$

$$R (\pi, 14)$$

- (a) By considering point P, the initial displacement is 14 cm.
- (b) By considering point Q, the pendulum returns to the middle position in  $\frac{\pi}{4}$  seconds.
- (c) By considering points P and R, the pendulum takes  $\pi$  seconds to complete a full swing.

(d) Consider equation in form:

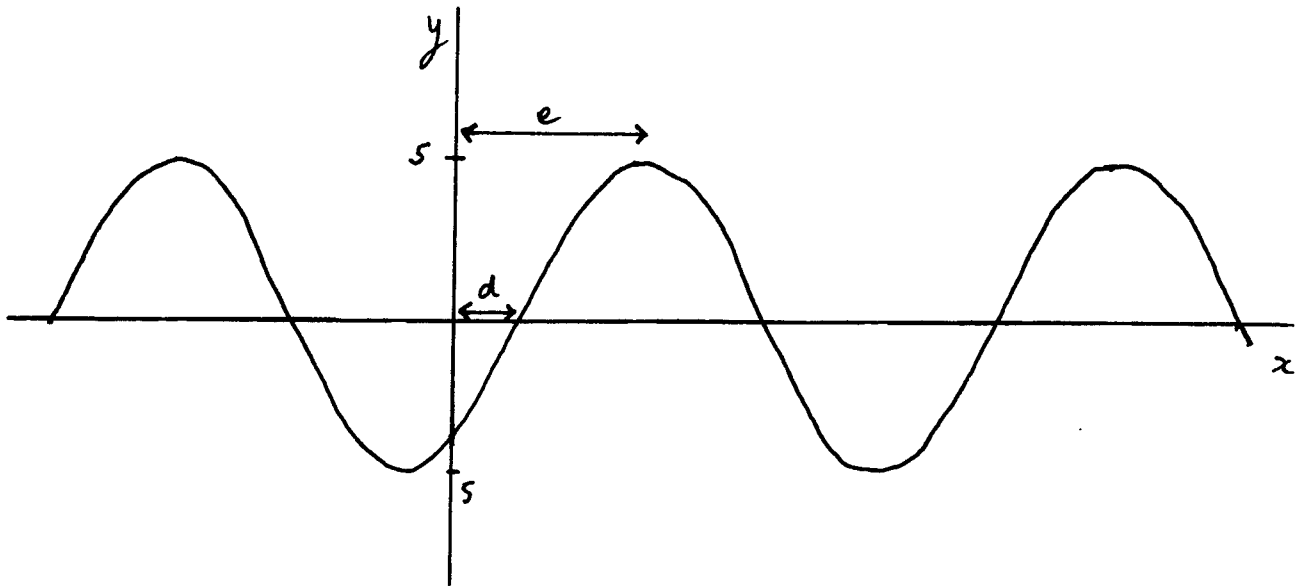
$$x(t) = a \cos \frac{2\pi}{\text{period}} (t - h) + k$$

$$a = 14, \text{ period} = 2\pi, h = 0, k = 0$$

$$\therefore x(t) = 14 \cos \frac{2\pi}{\pi} t$$

$$x(t) = 14 \cos 2t$$

$$(3) \quad y = 3 \sin x - 4 \cos x$$



from graph on TI-83 (zoom in) :

$$\text{amplitude} = 5$$

$$d \approx 0.93$$

$$e \approx 2.50$$

∴ The equation can be written in the following forms by considering horizontal shifts to the right.

$$y = 5 \sin (x - 0.93)$$

$$y = 5 \cos (x - 2.50)$$

Inspection of the graph shows that the period is  $2\pi$ .

$$2\pi - 0.93 \approx 5.35$$

$$2\pi - 2.50 \approx 3.78$$

∴ The equation can be written in the following forms by considering horizontal shifts to the left.

$$y = 5 \sin (x + 5.35)$$

$$y = 5 \cos (x + 3.78)$$