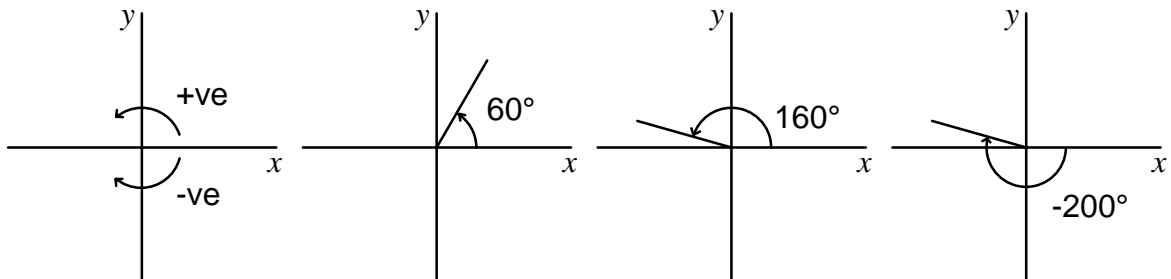


TRIGONOMETRIC RATIOS FOR ANY ANGLE

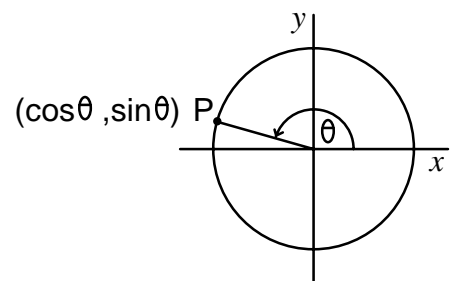
To decide what is meant by trigonometric ratios for all angles (not just acute), a different approach is required.

On the Cartesian plane, angles are measured *from the positive x-axis with anticlockwise* taken as *positive* as shown below. NB. The angles -200° and 160° are represented by the same line segment.

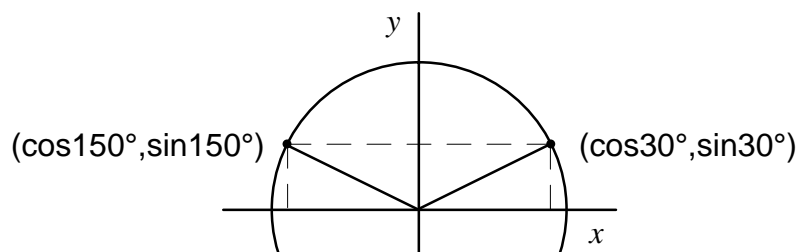


To define trigonometric ratios for any angle θ :

- Draw the *unit circle* (radius 1 and centre the origin).
- Draw the angle.
- Consider the point P on the unit circle.
- $\cos \theta$ is the x-coordinate of P.
- $\sin \theta$ is the y-coordinate of P.
- $\tan \theta = \frac{\sin \theta}{\cos \theta}$



Consider the angles 30° and 150° and the points on the unit circle.



Comparing x-coordinates: $\cos 150^\circ = -\cos 30^\circ$

Comparing y-coordinates: $\sin 150^\circ = \sin 30^\circ$

From the definition of tangent: $\tan 150^\circ = \frac{\sin 150^\circ}{\cos 150^\circ} = \frac{\sin 30^\circ}{-\cos 30^\circ} = -\frac{\sin 30^\circ}{\cos 30^\circ} = -\tan 30^\circ$

This is an example of a general result which works for all angles θ :

$\cos(180^\circ - \theta) = -\cos \theta$	$\sin(180^\circ - \theta) = \sin \theta$	$\tan(180^\circ - \theta) = -\tan \theta$
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