$$\int_a^{a+2h} x^3 \, dx$$

$$= \left[\tfrac{1}{4} x^4 \right]_a^{a+2h}$$

$$= \tfrac{1}{4}(a+2h)^4 - \tfrac{1}{4}a^4$$

$$= \tfrac{1}{4}\left(a^4 + 4a^3 . 2h + 6a^2 . (2h)^2 + 4a . (2h)^3 + (2h)^4\right) - \tfrac{1}{4}a^4$$

$$= 2a^3h + 6a^2h^2 + 8ah^3 + 4h^4$$

$$\tfrac{1}{3}(y_0 + 4y_1 + y_2)h$$

$$= \tfrac{1}{3}\left(a^3 + 4[a+h]^3 + [a+2h]^3\right) . h$$

$$= \tfrac{1}{3}\big(a^3 + 4[a^3 + 3a^2h + 3ah + h^3] + [a^3 + 3a^2 . 2h + 3a . (2h)^2 + (2h)^3]\big) . h$$

$$= \tfrac{1}{3}\left(a^3 + 4a^3 + 12a^2h + 12ah^2 + 4h^3 + a^3 + 6a^2h + 12ah^2 + 8h^3\right) . h$$

$$= \tfrac{1}{3}\left(6a^3 + 18a^2h + 24ah^2 + 12h^3\right) . h$$

$$= 2a^3h + 6a^2h^2 + 8ah^3 + 4h^4$$

$\therefore$ Simpson's Rule gives the exact result.